18th Junior Balkan Mathematical Olympiad
June 21-26, 2014, Ohrid, Republic of Macedonia

Language: *English*
Monday, June 23, 2014.

1. Find all distinct prime numbers p, q and r such that
$$3p^4 - 5q^4 - 4r^2 = 26.$$

2. Consider an acute triangle ABC with area S. Let $CD \perp AB$ ($D \in AB$), $DM \perp AC$ ($M \in AC$) and $DN \perp BC$ ($N \in BC$). Denote by H_1 and H_2 the orthocenters of the triangles MNC and MND respectively. Find the area of the quadrilateral AH_1BH_2 in terms of S.

3. Let a, b, c be positive real numbers such that $abc = 1$. Prove that
$$\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2 \geq 3(a+b+c+1).$$
When does equality hold?

4. For a positive integer n, two players A and B play the following game: Given a pile of s stones, the players take turn alternatively with A going first. On each turn the player is allowed to take either one stone, or a prime number of stones, or a positive multiple of n stones. The winner is the one who takes the last stone. Assuming both A and B play perfectly, for how many values of s the player A cannot win?

Time: 4 hours and 30 minutes
Each problem is worth 10 points